

SEMESTRAL
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TRIGONOMETRÍA

Tema: Identidades
trigonométricas III

IDENTIDADES DEL ARCO TRIPLE

$$\operatorname{sen} 3\theta = 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$$

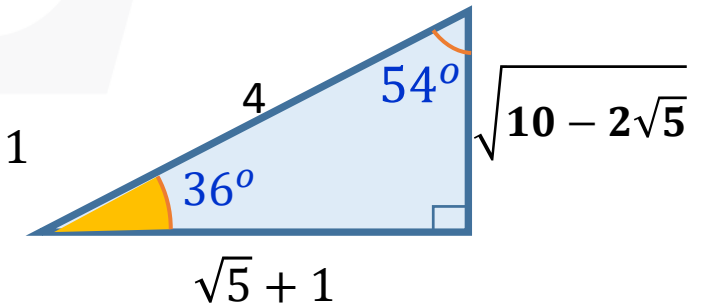
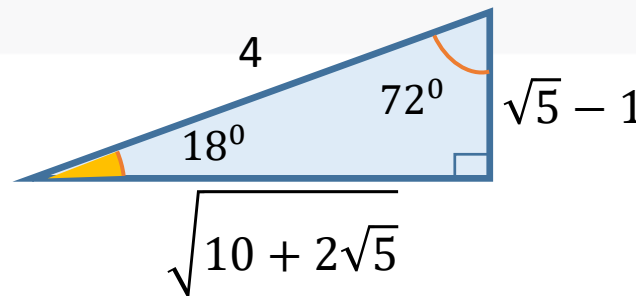
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

IDENTIDADES DE DEGRADACIÓN

$$4\operatorname{sen}^3\theta = 3\operatorname{sen}\theta - \operatorname{sen} 3\theta$$

$$4\cos^3\theta = 3\cos\theta + \cos 3\theta$$



Examen UNI 2006-I

Si

$$\text{sen}^3\left(\theta - \frac{\pi}{4}\right) + \text{sen}^3\theta + \text{sen}^3\left(\theta + \frac{\pi}{4}\right) \\ = a\text{sen}\theta + b\text{sen}3\theta$$

Determine $M = a - 3b$

- A) 1,5 B) 2,0 C) 2,5
D) 3,0 E) 3,5

RESOLUCIÓN

$$E = \text{sen}^3\left(\theta - \frac{\pi}{4}\right) + \text{sen}^3\theta + \text{sen}^3\left(\theta + \frac{\pi}{4}\right)$$

$$4E = 4\text{sen}^3\left(\theta - \frac{\pi}{4}\right) + 4\text{sen}^3\theta + 4\text{sen}^3\left(\theta + \frac{\pi}{4}\right)$$

$$4E = 3\text{sen}\left(\theta - \frac{\pi}{4}\right) - \text{sen}\left(3\theta - 3\frac{\pi}{4}\right) + 3\text{sen}\theta - \text{sen}3\theta + 3\text{sen}\left(\theta + \frac{\pi}{4}\right) - \text{sen}\left(3\theta + 3\frac{\pi}{4}\right)$$

$$4E = 3\left[\text{sen}\left(\theta - \frac{\pi}{4}\right) + \text{sen}\theta + \text{sen}\left(\theta + \frac{\pi}{4}\right)\right] - \left[\text{sen}\left(3\theta - 3\frac{\pi}{4}\right) + \text{sen}3\theta + \text{sen}\left(3\theta + 3\frac{\pi}{4}\right)\right]$$

$$E = \frac{3}{4}(\sqrt{2} + 1)\text{sen}\theta + \frac{1}{4}(\sqrt{2} - 1)\text{sen}3\theta$$

Comparando: $a = \frac{3}{4}(\sqrt{2} + 1)$ y $b = \frac{1}{4}(\sqrt{2} - 1)$

$$\therefore M = a - 3b = 1,5$$

IDENTIDADES AUXILIARES DEL ARCO TRIPLE

$$\text{sen}3\theta = \text{sen}\theta(2\cos2\theta + 1)$$

$$\text{sen}3\theta = 4\text{sen}\theta\text{sen}(60^\circ - \theta)\text{sen}(60^\circ + \theta)$$

$$\cos3\theta = \cos\theta(2\cos2\theta - 1)$$

$$\cos3\theta = 4\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta)$$

$$\tan3\theta = \tan\theta \left(\frac{2\cos2\theta + 1}{2\cos2\theta - 1} \right)$$

$$\tan3\theta = \tan\theta\tan(60^\circ - \theta)\tan(60^\circ + \theta)$$

Examen UNI 2013-II

Calcule el resultado simplificado de la siguiente expresión

$$E = 2^5 \operatorname{sen} 5^\circ \operatorname{sen} 10^\circ \operatorname{sen} 50^\circ \operatorname{sen} 70^\circ \operatorname{sen} 85^\circ \operatorname{sen} 110^\circ \operatorname{sen} 130^\circ$$

$$A) \frac{1}{4}$$

$$B) \frac{1}{2}$$

$$C) 1$$

$$D) 2$$

$$E) 4$$

RESOLUCIÓN

$$E = 16(2) \operatorname{sen} 5^\circ \operatorname{sen} 10^\circ \operatorname{sen} 50^\circ \operatorname{sen} 70^\circ \cos 5^\circ \operatorname{sen} 70^\circ \operatorname{sen} 50^\circ$$

$$E = 16(2 \operatorname{sen} 5^\circ \cos 5^\circ) \operatorname{sen} 10^\circ \operatorname{sen}^2 50^\circ \operatorname{sen}^2 70^\circ$$

$$E = 16 \operatorname{sen} 10^\circ \operatorname{sen} 10^\circ \operatorname{sen}^2 50^\circ \operatorname{sen}^2 70^\circ$$

$$E = 4^2 \operatorname{sen}^2 10^\circ \operatorname{sen}^2 50^\circ \operatorname{sen}^2 70^\circ$$

$$E = (4 \operatorname{sen} 10^\circ \operatorname{sen} 50^\circ \operatorname{sen} 70^\circ)^2$$

$$E = (4 \operatorname{sen} 10^\circ \operatorname{sen}(60^\circ - 10^\circ) \operatorname{sen}(60^\circ + 10^\circ))^2$$

$$E = (\operatorname{sen} 3(10^\circ))^2$$

$$E = (\operatorname{sen} 30^\circ)^2$$

$$E = \left(\frac{1}{2}\right)^2$$

$$\therefore E = \frac{1}{4}$$

IDENTIDADES TRIGONOMÉTRICAS DE TRANSFORMACIÓN**DE SUMA O DIFERENCIA A PRODUCTO**

$$\text{sen}A + \text{sen}B = 2\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{sen}A - \text{sen}B = 2\cos\left(\frac{A+B}{2}\right)\text{sen}\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\text{sen}\left(\frac{A+B}{2}\right)\text{sen}\left(\frac{A-B}{2}\right)$$

DE PRODUCTO A SUMA O DIFERENCIA

$$2\text{sen}x\cos y = \text{sen}(x+y) + \text{sen}(x-y)$$

$$2\cos x\cos y = \cos(x+y) + \cos(x-y)$$

$$2\text{sen}x\text{sen}y = \cos(x-y) - \cos(x+y)$$

Examen UNI 2010-II

Calcule el valor de:

$$E = \frac{1}{2\operatorname{sen}10^\circ} - 2\operatorname{sen}70^\circ$$

A) -1 B) 0 C) 1

D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{2}$

RESOLUCIÓN

$$E = \frac{1 - 2(2\operatorname{sen}70^\circ\operatorname{sen}10^\circ)}{2\operatorname{sen}10^\circ}$$

$$E = \frac{1 - 2(\cos60^\circ - \cos80^\circ)}{2\operatorname{sen}10^\circ}$$

$$E = \frac{1 - 2\left(\frac{1}{2} - \cos80^\circ\right)}{2\operatorname{sen}10^\circ}$$

$$= \frac{1 - 1 + 2\cos80^\circ}{2\operatorname{sen}10^\circ}$$

$$E = \frac{2\cos80^\circ}{2\operatorname{sen}10^\circ}$$

$$\therefore E = 1$$



GRACIAS

SÍGUENOS:   

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